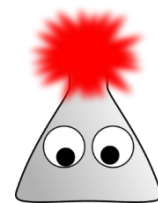


Significant Figures

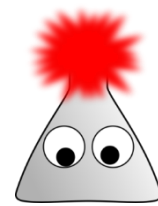
Reporting numbers with (un)certainty



Significant Figures (“Sig Figs”)

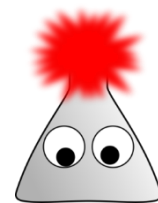
“Significant Figures” refers to the number of digits of information we can reasonably report for a given value based upon our level of uncertainty in the precision of that value.

Sig figs become important when we round numbers. By analyzing sig figs, we can determine where to round a number that is the result of a mathematical operation or series of operations.



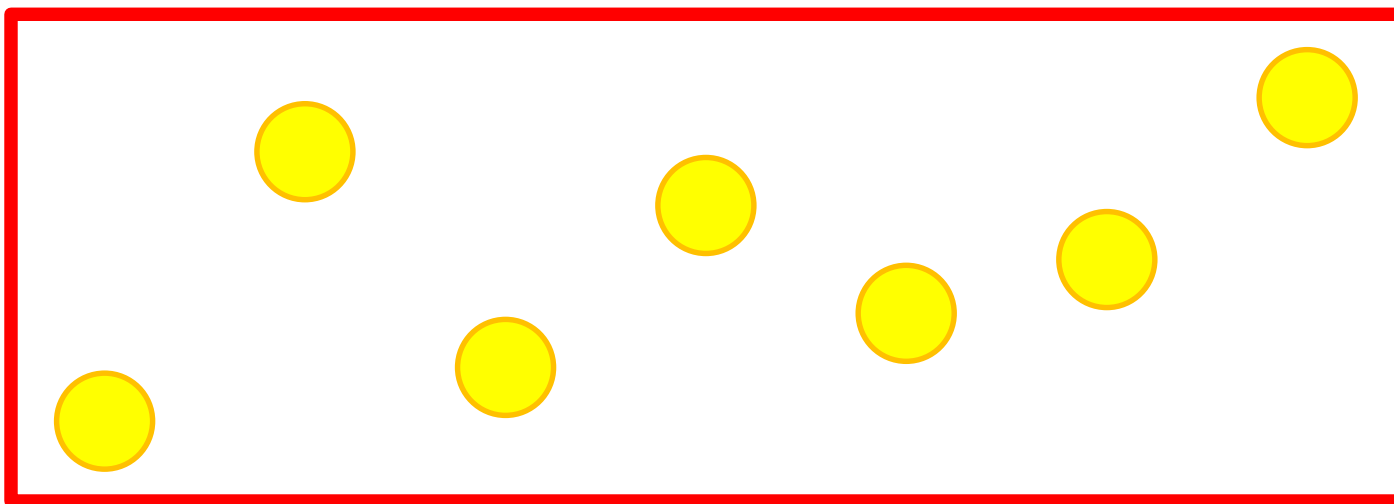
What does the number *mean*??

What does it mean when we report a number? To some extent, that depends on what the number represents. Let's start with "easy" numbers...



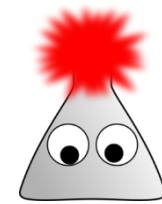
Counting Numbers

How many circles are in the box?



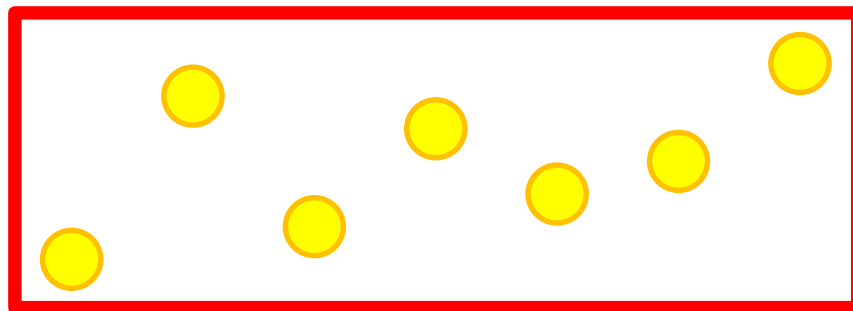
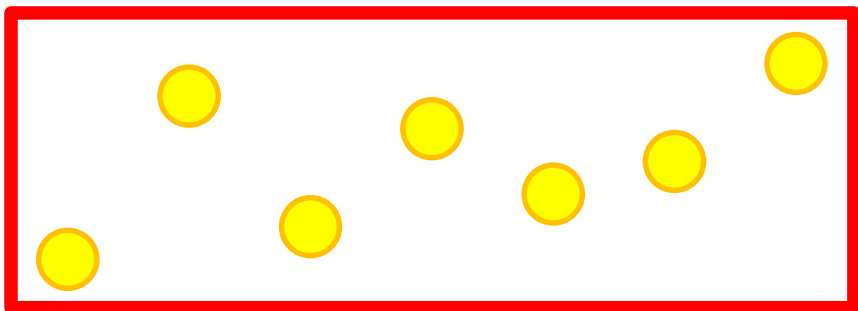
This is a counting exercise. There are exactly 7 circles in the box.

Numbers like this are infinitely precise, they do not limit the precision of our answer.



Math on counting numbers

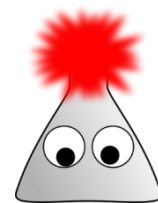
What if we have 2 boxes? How many circles?



OK, that's easy math. Two boxes (that's another "counting number" ...) have:

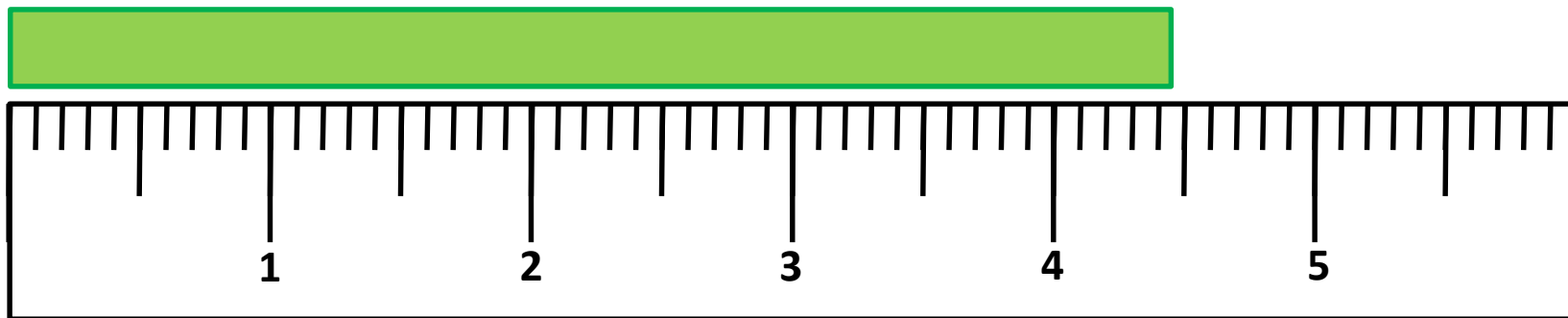
$$(2 \text{ boxes}) \times (7 \text{ circles per box}) = 14 \text{ circles}$$

That is a precise number; it's exactly 14.



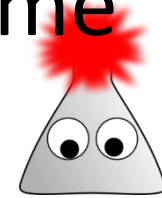
Imprecise (uncertain) numbers...

How long is the green bar?



It's more than 4 units, but less than 5. It looks like it's actually more than 4.4 units, but less than 4.5. I might call it 4.46 units.

The "4.4" part are certain digits. That last digit (the "6") is an estimate meaning there is some uncertainty in that digit, it could be 5 or 7.



Math on imprecise numbers

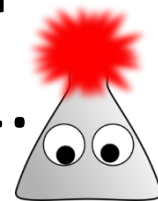
How long are *two* green bars?



We could stack them and use a longer ruler, but let's use math:

$$(2 \text{ bars}) \times (4.46 \text{ units per bar}) = 8.92 \text{ units}$$

Seems like the “circles in a box” example, but now we have uncertainty in our answer. My input (“4.46”) has uncertainty in the second decimal place, so my answer rounds to 8.92.



Significant Figures

The number of significant figures in a reported value is all of the certain digits plus one uncertain digit. It's probably easier to show than explain...

27.195

4 certain digits
1 uncertain digit



5 sig figs

9821.6

4 certain digits
1 uncertain digit



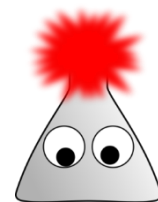
5 sig figs

0.33842

4 certain digits
1 uncertain digit



5 sig figs



What about zeros????

Zeros might be certain or they might be uncertain. So they might be sig figs. Or not...

19.063

Zeros *between* certain digits are certain.

4 certain digits
1 uncertain digit



5 sig figs

0.0063

Leading zeros are just place holders, they are not sig figs.

1 certain digits
1 uncertain digit



2 sig figs

28500

Trailing zeros are also just place holder so they are not sig figs.

2 certain digits
1 uncertain digit



3 sig figs

19.490

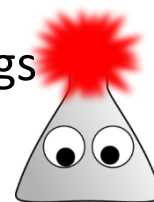
Trailing zeros after the decimal are sig figs because otherwise they wouldn't be written.

4 certain digits
1 uncertain digit



5 sig figs

BUT...

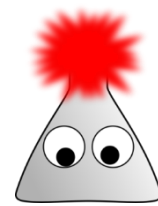


Are those *really* trailing zeros?

What if we have a number like “30”? It could be a counting number 30 (*infinitely precise*) or it could be a rounded number 30 (*1 or 2 sig fig*).

How do we know?!?!?!?

We need context. If we want to *report* that number, one way we can specify that the trailing zero *is* significant (*and therefore this is a 2 sig fig number*) is by writing it “30.” The decimal place implies that the zero is a sig fig.



OK, but trailing zeros again...

What if a number has some zeros that are sig figs and some that aren't? Something like:

210000

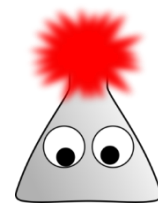
Drawing boxes makes it clear that "210" are certain digits and the next "0" is the uncertain digit, with 2 trailing zeros that are not sig figs.

3 certain digits
1 uncertain digit



4 sig figs

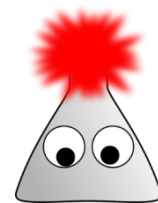
Drawing those boxes can be kind of awkward and requires explaining. The easiest way to express a number like this is using scientific notation: 2.100×10^5 . This way, the sig fig zeros are clearly shown.



Sig Fig Math Rules

When we do math, we have to make sure that we're communicating a number that accurately represents the uncertainty in our result.

There are a bunch of rules for this and a lot of different ways to approach it. We will use significant figures because it's actually a relatively simple way to keep track of certainty.



Sig Figs – Adding & Subtracting

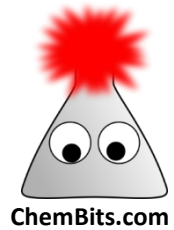
When adding & subtracting, the result should be rounded to the least precise position of the inputs.

$$\begin{array}{r} 97.35 \\ + 8.2273 \\ \hline 105.5773 \end{array}$$

$$\begin{array}{r} 97.35 \\ + 8.2273 \\ \hline 105.58 \end{array}$$

Round your answer to the least precise uncertain digit in your inputs

Line up the decimal points



Sig Figs – Multiplying & Dividing

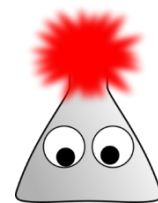
When multiplying & dividing, the result should be rounded to the same *number of sig figs* as the least precise input.

$$\begin{array}{r} 12.31 \\ \times 14.872 \\ \hline 183.1 \end{array}$$

Diagram illustrating the multiplication of 12.31 and 14.872. The result is 183.1. The number of significant figures (sig figs) is indicated for each number and the result:

- 12.31 has 4 sig figs
- 14.872 has 5 sig figs
- The result, 183.1, has 4 sig figs.

“12.31” has fewer sig figs than “14.872”, so the result is rounded to the same number of sig figs as “12.31” has, 4

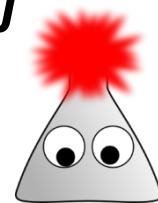


Sig Figs – Mixed operations

What if we have to add/subtract AND multiply/divide in the same problem? Apply the same rules for sig figs in the order that you perform the operations:

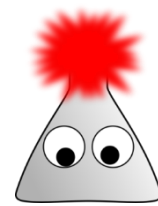
$$\underbrace{(17.3 \times 2.1334)}_{(36.90782)} + 6.58129 = 43.5$$
$$(36.90782) + 6.58129 = 43.5$$

For multi-part calculations, don't actually round any numbers off until you get to the final answer.



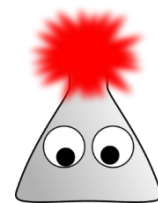
Sig Figs – What's the point?

Sig figs may seem a little picky and hard to follow, but the reason we use them is to clearly communicate what we mean when we report a number. Just like the sentence “She did it.” doesn't really communicate clearly because we don't know who “she” is or what “it” is without some additional context, numbers need some context and their own form of grammar to clearly communicate.



Practice, practice, practice

As with everything, sig figs get easier with practice. Whenever you're dealing with a number, think about which digits are certain and which digits are uncertain, and how that certainty or uncertainty interacts with other numbers around it.



Sig Figs – A practical example

You are purchasing small steel ball bearings that each weigh 0.0825g. You place 2 scoops of ball bearings in a bag to buy. At the checkout, the contents of the bag weigh 2.3kg. How many ball bearings are you buying?

$$(2.3\text{kg}) \left(\frac{1000\text{g}}{1\text{kg}} \right) \left(\frac{1 \text{ ball bearing}}{0.0825\text{g}} \right) = 27878.788 \text{ ball bearings}$$

1. How do you get 0.788 of a ball bearing?
2. Given the limited sig figs in “2.3kg”, you probably shouldn’t be overly confident that you have 27879 ball bearings... report this number as 28000.

